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CREEP DAMAGE UNDER ONE-DIMENSIONAL VARIABLE TENSILE STRESS*

Summary rept.,

by

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33

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ABSTRACT

A nonlinear damage relation, containing the axial strain history and a time integral over the stress history, is proposed for the case of one-dimensional time dependent tensile stress. Nonlinear steady and transient creep terms are included in the axial strain relation, and elastic and creep Poisson's ratios are introduced into the lateral strain relation. Using these relations, complete damage solutions are obtained for the constant stress rate, step stress, relaxation and constant load tests. Observations are made concerning the associated rupture times.

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1. INTRODUCTION

The problem of creep damage and rupture is currently of great practical importance, especially in the field of nuclear safety. This subject is very complex and much remains to be accomplished in the development of a complete and useful creep mechanics model for the estimation of rupture time. The publication based on the Nov. 1978 Ispra course on creep [1] contains detailed discussion on every aspect of this subject as well as an exhaustive bibliography; also see Hult [2] and the books by Hult [3] and Odqvist [4]. In a recent contribution to the subject of creep rupture, Belloni, Bernasconi and Piatti [5] have employed material density variation as a measure of damage in steel under constant one-dimensional tensile load. By extensive data analysis they found that the damage depends on the level of the initial stress, the time explicitly and also on the axial creep strain as a function of time. By employing a damage relation expressed in terms of power functions in these three variables and an axial creep strain relation consisting simply of a stress power law for steady creep, they were able to estimate the creep rupture time and thereby assign a physical interpretation (i.e., in terms of density variation) to the Kachanov rupture parameter (see [6]). Further results on this approach to creep damage were presented at two recent congresses [7,8].

In this paper we discuss in detail an extension of the damage relation presented in [5,7,8] to the case of time dependent one-dimensional tensile stress. We first modify this relation by stipulating that it be employed for constant stress rather than constant load,

and then postulate that for variable stress the damage rate be a function of the axial creep strain rate, axial creep strain, damage and stress. Also, we employ a fairly general axial strain relation containing linear elastic, nonlinear transient creep and nonlinear steady creep terms. The temperature is assumed to be constant in both the damage and strain constitutive relations. Using these relations, damage solution are then obtained for several prescribed one-dimensional stress and strain histories as well as for the case of constant load accompanied by lateral contraction. Such solutions are useful in the design and analysis of tensile test programs, and also for the estimation of rupture times.

Section 2 contains the one-dimensional model for damage and strain (both axial and lateral), first for constant stress and then as extended to variable stress. In Section 3 we obtain complete damage solutions for the constant stress rate test, the step stress test and the relaxation test with transient creep excluded in this last case. Solutions are then obtained in Section 4 for the constant load test with lateral contraction, first for large deformation with transient creep excluded (or included approximately in the elastic term) and a second for small deformation including transient creep as a separate term. A short discussion containing some suggestions for possible testing programs then concludes the paper.

2. CREEP MODEL

2.1 Damage Relation

For constant one-dimensional tensile stress at constant temperature we employ the damage relation

$$D(t) = C\ \varepsilon_c(t)^{\alpha}\ \sigma_o^{\gamma}\ t^{\delta} \qquad (1)$$

Here, $D(t)$ and $\varepsilon_c(t)$ are the damage and axial creep strain respectively at time t, σ_o is the constant stress, and $C, \alpha, \gamma, \delta$ are positive material damage constants at a particular temperature. In [5] σ_o was the initial stress for constant load and the damage $D(t)$ was given as $-\Delta\rho/\rho_o$ where $\Delta\rho$ is the density change $\rho(t) - \rho_o$ (negative) due to void formation and growth; here $D(t)$ is not necessarily restricted to this particular measure of damage. Equation (1) may also be written as

$$\frac{d}{dt}\left[\frac{D(t)}{C\ \varepsilon_c(t)^{\alpha}}\right]^{1/\delta} = \sigma_o^{\gamma/\delta} \qquad (2)$$

Note that $\varepsilon_c(t)$ does not include the elastic strain, which is consistent with the view that damage (such as due to void formation and growth) is not associated with recoverable deformation.

In generalizing eq. (1) to the case of variable stress $\sigma(t)$, we postulate that the damage rate be a function of the axial creep strain rate, axial creep strain, damage and stress, i.e.

$$\dot{D}(t) = f[\dot{\varepsilon}_c(t),\ \varepsilon_c(t),\ D(t),\ \sigma(t)] \qquad (3)$$

where the dot above a variable indicates d/dt. An expression consistent with this function is obtained directly if we simply replace σ_o in eq. (2) by $\sigma(t)$, which upon integration yields the desired result

$$D(t) = C\, \varepsilon_c(t)^{\alpha} \left[\int_o^t \sigma(t')^{\gamma/\delta} dt' \right]^{\delta} \tag{4}$$

For $\sigma(t) = \sigma_o$ eq. (4) clearly simplifies to eq. (1), and furthermore it possesses the physically necessary characteristic that $D(t)$ be continuous even if the stress history is discontinuous.

2.2 Strain Relations

For the axial creep strain at constant tensile stress and temperature appearing in Eq. (1), we employ in this paper the sum of axial creep strain components

$$\varepsilon_c(t) = \varepsilon_s(t) + \varepsilon_t(t) \tag{5}$$

where

$$\varepsilon_s(t) = \left(\frac{\sigma_o}{\lambda}\right)^n t, \quad \varepsilon_t(t) = \left(\frac{\sigma_o}{\mu}\right)^q (1 - e^{-t/\tau}) \tag{6}$$

Here, $\varepsilon_s(t)$ and $\varepsilon_t(t)$ represent the axial strains due to steady creep and transient creep respectively, and λ, n, μ, q, τ are positive material creep constants. Expressions of this type have been widely used in the study of creep in various metals and polymers, including the practically important case of stainless steel (see Garofalo [9]).

The power form in time is also widely used, but for the sake of brevity we restrict our attention here to strain in the form of eqs. (5-6).

In generalizing eqs. (6) to variable stress $\sigma(t)$ we follow the strain-hardening approach of Cozzarelli and Shaw [10], whereby each component of creep strain rate is expressed as a function of the same component of creep strain and the stress, i.e.

$$\dot{\varepsilon}_s(t) = g[\varepsilon_s(t), \sigma(t)], \quad \dot{\varepsilon}_t(t) = h[\varepsilon_t(t), \sigma(t)] \tag{7}$$

After appropriate differentiation and manipulation, one then obtains the desired generalizations of eqs. (6) in integral form as

$$\varepsilon_s(t) = \int_0^t \left(\frac{\sigma(t')}{\lambda}\right)^n dt', \quad \varepsilon_t(t) = \frac{1}{\tau} e^{-t/\tau} \int_0^t \left(\frac{\sigma(t')}{\mu}\right)^q e^{t'/\tau} dt' \tag{8}$$

Thus, the axial creep strain for use in eq. (4) is given by eq. (5) with eqs. (8).

In order to obtain the solutions in the next two sections, we shall also require expressions for the total axial strain and the total lateral strain. For the total axial strain $\varepsilon(t)$ we simply add a linear elastic term to the axial creep strain, i.e.

$$\varepsilon(t) = \frac{\sigma}{E} + \varepsilon_c(t) \tag{9}$$

where E is the elastic modulus. And for the total lateral strain $\varepsilon_\ell(t)$ we follow the apporach of Courtine, Cozzarelli and Shaw [11], and describe lateral contraction by means of

$$\varepsilon_\ell(t) = -[\nu \frac{\sigma}{E} + \nu_s \varepsilon_s(t) + \nu_t \varepsilon_t(t)] \tag{10}$$

where ν is the usual linear elastic Poisson's ratio, while ν_s and ν_t are steady creep and transient creep Poisson's ratios respectively. For metals, ν_s is close to 1/2 (incompressible) whereas ν_t is between ν and ν_s. Note that we have not employed above the simplified approach of combining elastic and transient creep strain into a single nonlinear instantaneous term (e.g., see [4]).

2.3 Rupture Time

The presence of the creep strain in the damage relation (4) was motivated by the experimental observations in [5,7,8], and results in a coupling of the damage and creep strain relations which is necessary for a physically reasonable estimation of rupture time. We have not followed Kachanov's approach [6] of including the damage in the creep strain relation, since it was felt that this additional coupling would complicate the formulation unnecessarily. Therefore the present formulation does not account for tertiary creep at constant stress, but this is not necessarily a restriction if for example one defines the rupture time as that time at which rapid tertiary creep and rupture commence. Furthermore, the significance of tertiary creep up to the point of actual rupture can be minimized if care is taken to base the creep model on constant stress data (as done here) rather than on constant load data.

Specifically, let t_r be the rupture time corresponding to a critical value of damage for rupture D_r. For the case of constant stress eqs. (1), (5) and (6) then yield

$$D_r = C\left[\left(\frac{\sigma_o}{\lambda}\right)^n t_r + \left(\frac{\sigma_o}{\mu}\right)^q (1 - e^{-t_r/\tau})\right]^\alpha \sigma_o^\gamma\, t_r^\delta \tag{11}$$

where gives the rupture time in implicit form. If we ignore transient creep $(\mu \to \infty)$ we may obtain t_r explicitly as

$$t_r = \left[\left(\frac{D_r \lambda^{n\alpha}}{C}\right)\sigma_o^{-(n\alpha+\gamma)}\right]^{\frac{1}{\alpha+\delta}} \tag{12}$$

which agrees with the result given in [5], except that there σ_o was the initial stress for constant load. It was also pointed out in [5] that the positive quantity $(n\alpha+\gamma)/(\alpha+\delta)$ is analagous to the exponent of stress (generally designated as ν) in the Kachanov damage law. In the next two sections we examine several more complicated tests.

3. DAMAGE SOLUTIONS FOR VARIOUS PRESCRIBED STRESSES AND STRAINS

Some insight into the physical nature of the proposed creep model may be gained by obtaining explicit expressions for the damage $D(t)$ with various prescribed stress and strain histories. Thus, in this section we obtain solutions to eq. (4) for the constant stress rate, step stress and relaxation tests, and also make some observations on the associated rupture times.

3.1 Constant Stress Rate Test

As a first example, consider the case of a stress increasing linearly with time from a zero value at $t = 0$, i.e.

$$\sigma(t) = k\, t\, H(t), \qquad k > 0 \tag{13}$$

where k is the constant stress rate and $H(t)$ is the unit step function. Restricting q to odd positive integers, eqs. (5) and (8) yield for this case the axial creep strain

$$\varepsilon_c(t) = \left[\left(\frac{k}{\lambda}\right)^n \frac{t^{n+1}}{n+1} + \left(\frac{k}{\mu}\right)^q \left(\sum_{i=o}^{q} \frac{(-1)^i \tau^i q!}{(q-i)!}\, t^{q-i} + \tau^q q!\, e^{-t/\tau}\right)\right] H(t) \tag{14}$$

Eq. (4) with eq. (13) then yields the damage in terms of $\varepsilon_c(t)$ as

$$\frac{D(t)}{C\varepsilon_c(t)^\alpha} = \left[\left(\frac{\delta}{\gamma+\delta}\right)^\delta k^\gamma\, t^{\gamma+\delta}\right] H(t) \tag{15}$$

If we ignore transient creep, eqs. (14) and (15) yield the simple expression for damage

$$D(t) = \frac{C}{(n+1)^{\alpha}} \left(\frac{\delta}{\gamma+\delta}\right)^{\delta} \lambda^{-n\alpha} k^{\gamma+n\alpha} t^{\gamma+\alpha(n+1)+\delta} H(t) \tag{16}$$

Note that in eqs. (15) and (16) $D(t) \to \infty$ as $t \to \infty$, and as in the constant stress test there will always exist a finite rupture time t_r at which the damage reaches a critical value D_r.

3.2 Step Stress Test

Now consider the case of a constant stress applied at $t = 0$ and then suddenly changed to $K\sigma_o$ at $t = t_1$, i.e.

$$\sigma(t) = \sigma_o \left\{ \left[H(t) - H(t - t_1) \right] + KH(t - t_1) \right\}, \qquad K > 0 \tag{17}$$

In this case, eqs. (5) and (8) give

$$\begin{aligned} \varepsilon_c(t) = {} & \left(\frac{\sigma_o}{\lambda}\right)^{n} [tH(t) + (K^n - 1)(t - t_1)H(t - t_1)] \\ & + \left(\frac{\sigma_o}{\mu}\right)^{q} [(1 - e^{-t/\tau})H(t) + (K^q - 1)(1 - e^{-(t-t_1)/\tau})H(t - t_1)] \end{aligned} \tag{18}$$

Next, eq. (14) gives

$$\frac{D(t)}{C\varepsilon_c(t)^{\alpha}} = \sigma_o^{\gamma}[t^{\delta}H(t) + (K^{\gamma}-1)(t^{\delta} - t_1^{\delta})H(t - t_1)] \tag{19}$$

Again, if we ignore transient creep a simplified expression for damage is obtained as

$$D(t) = C\,\lambda^{-\alpha n}\,\sigma_o^{\gamma+n\alpha}\Big\{t^{\delta+\alpha}\,H(t) + [(K^{\gamma}-1)\;t^{\alpha}(t^{\delta}-t_1^{\delta})$$

$$+(K^{\alpha n}-1)\;t^{\delta}(t^{\alpha}-t_1^{\alpha})+(K^{\gamma}-1)(K^{\alpha n}-1)(t^{\delta}-t_1^{\delta})(t^{\alpha}-t_1^{\alpha})]H(t-t_1)\Big\} \tag{20}$$

As in the constant stress rate test, $D(t) \to \infty$ as $t \to \infty$ in the present case and a finite rupture time t_r again exists. Also note that although in accordance with eq. (17) $\sigma(t)$ is discontinuous at $t = t_1$, the damage $D(t)$ in accordance with eqs. (19) and (18) is continuous at this point while $\dot{D}(t_1)$ *is discontinuous.*

3.3 Relaxation Test

As a third example, consider the case of a constant strain suddenly applied at $t = 0$, i.e.

$$\varepsilon(t) = \varepsilon_o\,H(t) \tag{21}$$

When $\varepsilon(t)$ is prescribed, eq. (9)(with eqs. (5) and (8)) is a nonlinear differential equation in $\sigma(t)$ which may in general be difficult to solve. However, if we ignore transient creep, this differential equations *with condition (21) simplifies to*

$$\frac{1}{E}\frac{d\sigma}{dt} + \left(\frac{\sigma}{\lambda}\right)^{n} = 0\;, \qquad t > 0 \tag{22}$$

where $\sigma(0^+) = E\,\varepsilon_o$. For $n > 1$, the solution to eq. (22) is readily obtained as

$$\sigma(t) = E\left[\varepsilon_o^{-n+1} + (n-1)\;E^{n}\lambda^{-n}\;t\right]^{\frac{1}{-n+1}} H(t) \tag{23}$$

The relaxation test with transient creep neglected is thus equivalent to a test where stress is prescribed in accordance with eq. (23). Substituting eqs. (21) and (23) into eq. (9) we then obtain the creep strain as

$$\varepsilon_c(t) = \left\{\varepsilon_o - [\varepsilon_o^{-n+1} + (n-1)E^n \lambda^{-n} t]^{\frac{1}{-n+1}}\right\} H(t) \tag{24}$$

Finally, eq. (4) with eq. (23) yields the damage in terms of $\varepsilon_c(t)$ as

$$\frac{D(t)}{C\varepsilon_c(t)^\alpha} = \left(\frac{E^{-1}\lambda^n \delta}{\delta(n-1)-\gamma}\right)^\delta \left\{[(E\varepsilon_o)^{-n+1} + (n-1)E\lambda^{-n} t]^{\frac{\delta(n-1)-\gamma}{\delta(n-1)}} - [(E\varepsilon_o)^{-n+1}]^{\frac{\delta(n-1)-\gamma}{\delta(n-1)}}\right\}^\delta H(t) \tag{25}$$

In the limit as $t \to \infty$ we obtain from eqs. (24) and (25)

$$\lim_{t\to\infty} D(t) = \begin{cases} C\varepsilon_o^{\alpha+\gamma-\delta(n-1)} E^{\gamma-\delta n}\left(\dfrac{\lambda^n \delta}{\gamma-\delta(n-1)}\right)^\delta , & \omega > 0 \\ \infty , & \omega < 0 \end{cases} \tag{26a}$$

$$\text{with} \quad \omega = \gamma - \delta(n-1) \tag{26b}$$

We thus obtain the interesting result that whereas for the material power combination $\omega > 0$ the damage may not necessarily ever obtain a critical value D_r, the existence of a finite rupture time t_r is guaranteed when $\omega < 0$. A similar result is obtained when one studies damage during stress relaxation using the Kachanov damage formulation.

We treat the constant load test separately in the next section, since that case is somewhat more complicated than the three cases considered in this section.

4. DAMAGE SOLUTIONS FOR CONSTANT LOAD

Consider now a bar of instantaneous cross-sectional area $A(t)$ under a constant load P_o suddenly applied at $t = 0$. The bar experiences a continuously increasing stress

$$\sigma(t) = \frac{P_o}{A(t)} \tag{27}$$

due to the effect of lateral contraction, and the existance of a finite rupture time is assured. Employing the logarithmic definition for large strain, we write the lateral strain rate as

$$\dot{\varepsilon}_\ell(t) = \frac{\dot{R}(t)}{R(t)} = \frac{1}{2}\frac{\dot{A}(t)}{A(t)} \tag{28}$$

where $R(t)$ is the instantaneous cross-sectional radius. In this section we determine the solutions for $A(t)$, $\sigma(t)$, $\varepsilon_c(t)$ and $D(t)$, first for the case of large deformation with transient creep excluded (or included approximately in the elastic term) and then for the case of small deformation with transient creep included.

4.1 Large Deformation - Transient Creep Excluded (or Included Approximately)

For this case we set $\varepsilon_t = 0$ in eq. (10) and also assume that the steady creep is incompressible ($\nu_s = 1/2$), whereupon with the use of eqs. (8) and (27) we obtain

$$\dot{\varepsilon}_\ell(t) = -\nu\,\dot{\overline{\left(\frac{P_o}{A(t)E}\right)}} - \frac{1}{2}\left(\frac{P_o}{A(t)\lambda}\right)^n , \quad t > 0 \tag{29}$$

This result in combination with eq. (28) leads to the differential equation in $A(t)$

$$\left(\frac{P_o}{\lambda}\right)^n dt = \frac{2\nu P_o}{E} A^{n-2} dA - A^{n-1} dA, \quad t > 0 \tag{30}$$

One may obtain a rough correction for the exclusion of transient creep by replacing E with E^*, a smaller fictitious modulus of elasticity. The initial condition for eq. (30) is prescribed at $t = 0^+$ (i.e., immediately after the load is applied), i.e.

$$A(0^+) = A_o \tag{31}$$

Since the strain at $t = 0^+$ is small we may write $\varepsilon_\ell(0^+) \simeq 1/2[A(0^+) - A(0^-)]/A(0^+)$ and thereby obtain

$$A(0^+) \simeq A(0^-) - \frac{2\nu P_o}{E} \tag{32}$$

from which we see that $A(0^+)$ differs from the known $A(0^-)$ by a very small amount.

Differential equation (30) with initial condition (31) may be integrated to yield

$$\left(\frac{A}{A_o}\right)^n \left[1 - \frac{2n\nu\sigma_o}{(n-1)E}\left(\frac{A}{A_o}\right)^{-1}\right] = 1 - \frac{2n\nu\sigma_o}{(n-1)E} - n\left(\frac{\sigma_o}{\lambda}\right)^n t, \quad t > 0 \tag{33}$$

where $\sigma_o = P_o/A_o$. Eq. (33) gives the function $A_o/A(t) = F(t)$ in implicit form, but explicit expressions may be obtained for approximate cases. For example, if we neglect the elastic term $(E \to \infty)$

eq. (33) gives the simple relation

$$\frac{A_o}{A(t)} \approx \left[1 - n\left(\frac{\sigma_o}{\lambda}\right)^n t\right]^{-\frac{1}{n}}, \qquad t > 0 \tag{34}$$

On the other hand, we can retain the elastic term, but approximate A_o/A within the brackets by unity, and accordingly obtain

$$\frac{A_o}{A(t)} \approx \left[1 - \frac{n(n-1)E}{(n-1)E-2n\nu\sigma_o}\left(\frac{\sigma_o}{\lambda}\right)^n t\right]^{-\frac{1}{n}}, \qquad t > 0 \tag{35}$$

Since eq. (35) contains eq. (34) as a special case, we shall use eq. (35) in the remainder of this subsection, with E replaced by E^* when transient creep is approximated.

The stress now follows immediately from eq. (35) as

$$\sigma(t) = \sigma_o \frac{A_o}{A(t)} \approx \sigma_o \left[1 - \frac{n(n-1)E}{(n-1)E-2n\nu\sigma_o}\left(\frac{\sigma_o}{\lambda}\right)^n t\right]^{-\frac{1}{n}}, \qquad t > 0 \tag{36}$$

Next, the creep strain as obtained by substituting eq. (36) into the first of eqs. (8) is given by

$$\epsilon_c(t) \approx \frac{-\ln\left[1- \frac{n(n-1)E}{(n-1)E-2n\nu\sigma_o}\left(\frac{\sigma_c}{\lambda}\right)^n t\right]}{\frac{n(n-1)E}{(n-1)E-2n\nu\sigma_o}}, \qquad t > 0 \tag{37}$$

Note that $\epsilon_c(t) \to \infty$ as $t \to t_{max}$, where

$$t_{max} = \frac{(n-1)E-2n\nu\sigma_o}{n(n-1)E}\left(\frac{\sigma_o}{\lambda}\right)^{-n} \tag{38}$$

which is analagous to the ductile rupture criterion due to Hoff [12].

Finally, the damage is obtained in terms of $\varepsilon_c(t)$ from eqs. (4) and (36) as

$$\frac{D(t)}{C\varepsilon_c(t)^{\alpha}} \approx \sigma_o{}^{\gamma}\left\{\left(\frac{\delta n}{\gamma-\delta n}\right) t_{max}\left[\left(1-\frac{t}{t_{max}}\right)^{\frac{\delta n-\gamma}{\delta n}}-1\right]\right\}^{\delta}, \quad t>0 \qquad (39)$$

Note that as expected a finite rupture time t_r exists corresponding to a given critical damage D_r , but now in contrast with the constant stress test [eqs. (11-12)] $D(t) \rightarrow \infty$ as $t \rightarrow t_{max}$. Also note that in general $\gamma \gtreqless \delta n$ in eq. (39), and in fact for the very special case $\gamma = \delta n$ eq. (39) simplifies to

$$\frac{D(t)}{C\varepsilon_c(t)^{\alpha}} \approx \sigma_o{}^{\gamma}\left[t_{max}\ \ln\left(1-\frac{t}{t_{max}}\right)\right]^{\delta} \qquad (40)$$

In order to illustrate the character of the above strain and damage solutions, we employ the data given in [8] for the following two steels:

material a: AISI 310 stainless steel at 600°C

material b: 2.25Cr 1Mo ferritic steel at 550°C

The various material parameters (elastic - E^*,ν; creep - n,λ^{-n}; damage $-\alpha,\gamma,\delta,C$) for these two materials are summarized in Table 1, where the E^* indicates we are using the rough correction for the exclusion of transient creep. Using these values with an initial stress σ_o equal to 30 kg/mm^2 for material a and 25 kg/mm^2 for material b, we obtained the constant load curves $\varepsilon_c(t)_{P_o}$ [eq. (37)] and $D(t)_{P_o}$ [eq. (39)] shown plotted in Figures 1 and 2 respectively vs. time in hrs. For purposes of comparison we have also shown in Figures 1 and 2 the constant stress curves $\varepsilon_c(t)_{\sigma_o}$ and $D(t)_{\sigma_o}$ [eqs. (1,5,6) with $\varepsilon_t = 0$] for these

same values of the material constants and σ_o . It is noteworthy that the curves at constant load differ very significantly from those at constant stress.

In Figure 1 we have indicated the ductile rupture times (t_{max}) corresponding to $\varepsilon_c \to \infty$. Also, in Figure 2 we have marked the rupture times at constant load and stress $[(t_r)_{P_o}, (t_r)_{\sigma_o}]$ corresponding to critical values of damage D_r equal to 25×10^{-4} for material a and 15×10^{-4} for material b (from [8]). These various rupture times are summarized in Table 2; as expected the rupture times at constant load are less than both the ductile rupture times and the rupture times at constant stress.

TABLE 1 - Material Constants

	Material	
	a	*b*
E^*, kg/mm^2	1000	1000
ν	.3	.3
n	7	5.6
λ^{-n}, mm^{2n}/kg^n-hr	0.74×10^{-13}	0.45×10^{-10}
α	.7	.65
γ	.06	2.5
δ	.012	.25
C, $mm^{2\gamma}/kg^{\gamma}$-hr^{δ}	4.3×10^{-3}	5.6×10^{-7}

TABLE 2 - Rupture Times

Constant Load and Stress - Large Deformation -

Transient Creep Included Approximately in E*

	Material	
	a	b
t_{max}, hr	86.4	57.8
$(t_r)_{P_o}$, hr	77.6	33.6
$(t_r)_{\sigma_o}$, hr	196.1	55.9

4.2 Small Deformation - Including Transient Creep as a Separate Term

Consider again the derivation of a differential equation in $A(t)$, except that now we retain the transient creep term $\varepsilon_t(t)$ and also do not restrict ν_s to 1/2. To this end we use eq. (28) to form the combination

$$\ddot{\varepsilon}_\ell(t) + \frac{1}{\tau}\dot{\varepsilon}_\ell(t) = \frac{1}{2}\dot{\overline{\left(\frac{\dot{A}}{A}\right)}} + \frac{1}{2\tau}\frac{\dot{A}}{A} \tag{41}$$

If we now substitute eq. (10) with eqs. (8) and (27) into the left hand side of eq. (41), we get the nonlinear differential equation

$$\frac{\nu P_o}{E}\ddot{B} + \frac{\nu P_o}{\tau E}\dot{B} + \nu_s\left(\frac{P_o}{\lambda}\right)^n \dot{\overline{B^n}} + \frac{\nu_s}{\tau}\left(\frac{P_o}{\lambda}\right)^n B^n + \frac{\nu_t}{\tau}\left(\frac{P_o}{\mu}\right)^q \dot{\overline{B^q}}$$

$$= \frac{1}{2}\dot{\overline{\left(\frac{\dot{B}}{B}\right)}} + \frac{1}{2\tau}\left(\frac{\dot{B}}{B}\right) \quad , \quad t > 0 \tag{42}$$

where $B(t) = 1/A(t)$.

Equation (42) cannot in general be solved in closed form, and thus we shall simplify the problem by introducing the assumption of small deformation. Accordingly, we write

$$A(t) = A_o - \tilde{A}(t) \ , \quad \tilde{A}(t) << A_o \tag{43}$$

where $\tilde{A}(t)$ is a small area increment and A_o has been defined by eq. (31) with eq. (32). Substituting eq. (43) into eq. (42) and neglecting higher order terms, we now obtain a linear differential equation in $A(t)$ as

$$\left(\frac{1}{2} - \frac{\nu\sigma_o}{E}\right)\ddot{\tilde{A}} + \left[\frac{1}{2\tau} - \frac{\nu\sigma_o}{\tau E} - n\nu_s \left(\frac{\sigma_o}{\lambda}\right)^n - \frac{q\nu_t}{\tau}\left(\frac{\sigma_o}{\mu}\right)^q\right]\dot{\tilde{A}}$$

$$- \frac{n\nu_s}{\tau}\left(\frac{\sigma_o}{\lambda}\right)^n \tilde{A} = \frac{A_o\nu_s}{\tau}\left(\frac{\sigma_o}{\lambda}\right)^n , \quad t > 0 \tag{44}$$

where as before $\sigma_o = P_o/A_o$.

For convenience we introduce the nondimensional variables

$$\bar{\tilde{A}} = \frac{\tilde{A}}{A_o} \ , \qquad \bar{t} = \frac{t}{\tau} \tag{45}$$

Equation (44) then becomes

$$a_1 \bar{\tilde{A}}'' + a_2 \bar{\tilde{A}}' - a_3 \bar{\tilde{A}} = a_3/n \ , \quad \bar{t} > 0 \tag{46}$$

where the prime indicates $d/d\bar{t}$, and the nondimensional coefficients are defined by

$$a_1 = \frac{1}{2} - \frac{\nu\sigma_o}{E} \tag{47a}$$

$$a_2 = \frac{1}{2} - \frac{\nu\sigma_o}{E} - n\nu_s\,\tau\left(\frac{\sigma_o}{\lambda}\right)^n - q\nu_t\left(\frac{\sigma_o}{\mu}\right)^q \tag{47b}$$

$$a_3 = n\nu_s\,\tau\left(\frac{\sigma_o}{\lambda}\right)^n \tag{47c}$$

As initial conditions for eq. (46) we require $\bar{\tilde{A}}(0^+)$ and $\bar{\tilde{A}}'(0^+)$. The former follows directly from eqs. (31), (43) and (45) as

$$\bar{\tilde{A}}(0^+) = 0 \tag{48}$$

In obtaining the latter we first note that $\varepsilon_s(0^+) = \varepsilon_t(0^+) = 0$, and use eqs. (8), (10) and (28) to obtain

$$\dot{\varepsilon}_\ell(0^+) = \frac{1}{2}\,\frac{\dot{A}(0^+)}{A_o} = \left(\frac{\nu P_o}{A_o^2 E}\right)\dot{A}(0^+) - \frac{\nu_t}{\tau}\left(\frac{\sigma_o}{\mu}\right)^q - \nu_s\left(\frac{\sigma_o}{\lambda}\right)^n \tag{49}$$

Introducing eqs. (43), (45) and (47) into this result and again assuming small deformation we obtain the desired condition as

$$\bar{\tilde{A}}'(0^+) = \frac{a_3}{a_1 n} + \frac{1}{q} - \frac{a_3}{a_1 q} - \frac{a_2}{a_1 q} \tag{50}$$

The solution to differential equation (46) with initial conditions (48) and (50) is easily obtained. For example, in the special case of $q = n$ we get

$$\begin{aligned}\bar{\tilde{A}}(\bar{t}) = -\frac{1}{n} &+ \left(\frac{-a_2 + 2a_1 + \Delta}{2n\Delta}\right)\exp\left(\frac{-a_2+\Delta}{2a_1}\right)\bar{t} \\ &+ \left(\frac{a_2 - 2a_1 + \Delta}{2n\Delta}\right)\exp\left(\frac{-a_2-\Delta}{2a_1}\right)\bar{t}\,, \quad \bar{t} > 0\end{aligned} \tag{51}$$

where

$$\Delta = (a_2^2 + 4\, a_1 a_3)^{1/2} \tag{52}$$

The stress then follows directly from eq. (51) in accordance with

$$\sigma(\bar{t}) = \sigma_o[1 + \bar{\bar{A}}(\bar{t})]\ , \quad \bar{t} > 0 \tag{53}$$

Substituting eq. (53) into eq. (5) with eq. (8) and neglecting higher order terms, we obtain the creep strain as

$$\begin{aligned}\varepsilon_c(\bar{t}) = &-\frac{a_1}{a_3}\,\tau\left(\frac{\sigma_o}{\lambda}\right)^n \\ &+ \left(\frac{-a_2 + 2a_1 + \Delta}{2\Delta}\right)\left[\tau\left(\frac{2a_1}{-a_2+\Delta}\right)\left(\frac{\sigma_o}{\lambda}\right)^n + \left(\frac{2a_1}{2a_1 - a_2 + \Delta}\right)\left(\frac{\sigma_o}{\mu}\right)^n\right]\exp\left(\frac{-a_2+\Delta}{2a_1}\,\bar{t}\right) \\ &+ \left(\frac{a_2 - 2a_1 + \Delta}{2\Delta}\right)\left[\tau\left(\frac{2a_1}{-a_2-\Delta}\right)\left(\frac{\sigma_o}{\lambda}\right)^n + \left(\frac{2a_1}{2a_1 - a_2 - \Delta}\right)\left(\frac{\sigma_o}{\mu}\right)^n\right]\exp\left(\frac{-a_2-\Delta}{2a_1}\,\bar{t}\right) \\ &\qquad \bar{t} > 0\end{aligned} \tag{54}$$

Although the creep model at constant stress (i.e., eqs. (5) and (6)) does not contain a tertiary creep stage, eq. (54) does yield a region of increasing creep rate after a critical time $\bar{t}_c$. Setting $\varepsilon_c''(\bar{t}_c) = 0$ we obtain this critical time as

$$\bar{t}_c = \frac{a_1}{\Delta}\ \ln\left\{\frac{(-a_2-\Delta)\left[(2a_1 - a_2 - \Delta)\,\tau\left(\frac{\sigma_o}{\lambda}\right)^n + (-a_2-\Delta)\left(\frac{\sigma_o}{\mu}\right)^n\right]}{(-a_2+\Delta)\left[(2a_1 - a_2 + \Delta)\,\tau\left(\frac{\sigma_o}{\lambda}\right)^n + (-a_2+\Delta)\left(\frac{\sigma_o}{\mu}\right)^n\right]}\right\} \tag{55}$$

Finally, the damage for small deformations is obtained from eq. (4) as

$$\frac{D(\bar{t})}{C\varepsilon_c(\bar{t})^{\alpha}} = \sigma_o^{\gamma}\left\{(1-\frac{\gamma}{\delta n})\tau\bar{t} - \frac{\gamma\tau}{\delta n}\frac{a_1}{a_3}\right.$$

$$+ \frac{\gamma\tau}{\delta}(\frac{-a_2+2a_1+\Delta}{2n\Delta})(\frac{2a_1}{-a_2+\Delta})\exp(\frac{-a_2+\Delta}{2a_1})\bar{t}$$

$$\left.+ \frac{\gamma\tau}{\delta}(\frac{a_2-2a_1+\Delta}{2n\Delta})(\frac{2a_1}{-a_2-\Delta})\exp(\frac{-a_2-\Delta}{2a_1})\bar{t}\right\}^{\delta} \qquad (56)$$

where we again find that t_r is finite.

In order to illustrate the above solutions the creep strain was calculated using the parameters assembled in [13] from the data of Garafalo et al. [9,14] for the following steel:

material d: AISI 316 stainless steel at 816°C

These elastic, steady creep and transient creep parameters $(E,n,\lambda^{-n},q,\mu^{-q},\tau)$ are given in Table 3 along with the additional assumed Poisson's ratios $\nu = .3$, $\nu_t = .4$ and $\nu_s = .5$. Figure 3 shows the ε_c curves for constant load [eq. (54)] and constant stress [eqs. (5,6)] for σ_o equal to 7.03 kg/mm^2, and we again see that these two solutions differ by a considerable amount. Upon comparing Figures 1 and 3 we see that the ductile rupture time t_{max} is no longer finite when deformations are small. However, we now have the critical time t_c [eq. (55)] corresponding to the point of inflection, and in the present example this occurred at 84 hours (see Figure 3).

TABLE 3 - Material Constants

	Material d
E, kg/mm^2	10,500
ν	.3
ν_t	.4
ν_s	.5
$n = q$	3.5
λ^{-n}, mm^{2n}/kg^n-hr	0.795×10^{-6}
μ^{-q}, mm^{2q}/kg^q	0.802×10^{-4}
τ , hr	21.7

5. DISCUSSION

Starting with a damage relation for constant one-dimensional tensile load given in [5], an extended damage relation was proposed for time dependent one-dimensional tensile stress histories. This relation contains a power of the strain history and also a power of a time integral over a power of the stress history, thereby ensuring a continuous damage history even for a discontinuous stress history. Employing an axial strain relation containing both steady and transient creep terms with a lateral strain relation containing three Poisson's ratios, damage solutions were obtained first for the constant stress rate, step stress and relaxation tests and then for the constant load test.

In all tests considered, except the relaxation test, there existed a finite rupture time corresponding to a critical value of damage; the

existence of a finite rupture time was guaranteed in the case of the relaxation test only when a material power combination was negative. Constant load test solutions were obtained both for the case of large deformation with transient creep excluded (or included approximately in the elastic terms) and for the case of small deformation with transient creep included as a separate term; in both cases the creep strain and damage solutions are considerably greater than the corresponding solutions at constant stress. In the former case a ductile rupture time was obtained corresponding to infinite strain. In the latter case a critical time was observed marking the beginning of a region of increasing creep rate.

A testing program for the determination of the material parameters would most logically begin with the one-dimensional constant tensile stress test, with the axial creep strain and damage measured as a function of time. Density variation is one useful index of damage, but it may be necessary to also use other methods such as the actual counting of voids. The most reliable procedure would probably be to first fit the axial creep strain data to determine the material parameters in the model for $\varepsilon_c(t)$, and then to smooth the damage data with the calculated $\varepsilon_c(t)$ before finally fitting this damage data to determine the material parameters in the model for $D(t)$. Then the validity of the proposed model for one-dimensional time dependent tensile stress could be checked by performing one or more of the tests discussed in this paper, with the step stress and constant load tests being possibly the most useful.

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APPENDIX: LIST OF SYMBOLS

a_1, a_2, a_3	nondimensional coefficients, eqs. (47)
$A(t)$	instantaneous circular cross-sectional area
A_o	initial cross-sectional area immediately after load application, eq. (31)
$\tilde{A}(t)$	small area increment, eq. (43)
$\bar{\tilde{A}}(t)$	nondimensional area, eq. (45)
$B(t)$	reciprocal of $A(t)$, eq. (42)
C	damage constant, eq. (1)
$D(t)$	damage, eq. (4)
D_r	critical value of damage for rupture
E	elastic modulus
E^*	fictitious elastic modulus to approximate transient creep
$H(t)$	unit step function
k	constant stress rate, eq. (13)
K	constant multiplier in step stress test, eq. (17)
n, q	creep constants, eq. (6)
P_o	constant load suddenly applied at $t = 0$
$R(t)$	instantaneous cross-sectional radius
t	time
t_{max}	ductile rupture time, eq. (38)
t_r	rupture time
t_1	constant time in step stress test, eq. (17)
$\bar{t}$	nondimensional time, eq. (45)
$\bar{t}_c$	critical time for constant load at beginning of period of increasing creep rate, eq. (55)

α,γ,δ	damage constants, eq. (1)
Δ	nondimensional constant, eq. (52)
$\varepsilon(t)$	total axial strain, eq. (9)
$\varepsilon_c(t)$	axial creep strain, eq. (5)
$\varepsilon_s(t),\varepsilon_t(t)$	axial strains due to steady creep and transient creep respectively, eqs. (6)
$\varepsilon_\ell(t)$	total lateral strain, eq. (10)
ε_o	constant strain in relaxation test, eq. (21)
λ,μ,τ	creep constants, eqs. (6)
ν	elastic Poisson's ratio
ν_s,ν_t	steadt creep and transient creep Poisson's ratios, eqs. (10)
$\rho(t)$	density
ρ_o	initial density
$\sigma(t)$	one-dimensional tensile stress
σ_o	constant stress
ω	material power combination in relaxation test, eq. (26b)
$(\dot{\ })$	indicates d/dt
$(\ )'$	indicates $d/d\bar{t}$
$(\ )_{P_o}$	indicates constant load test
$(\ )_{\sigma_o}$	indicates constant stress test
$(\ )_a,(\ )_b,(\ )_d$	indicates materials a,b and d

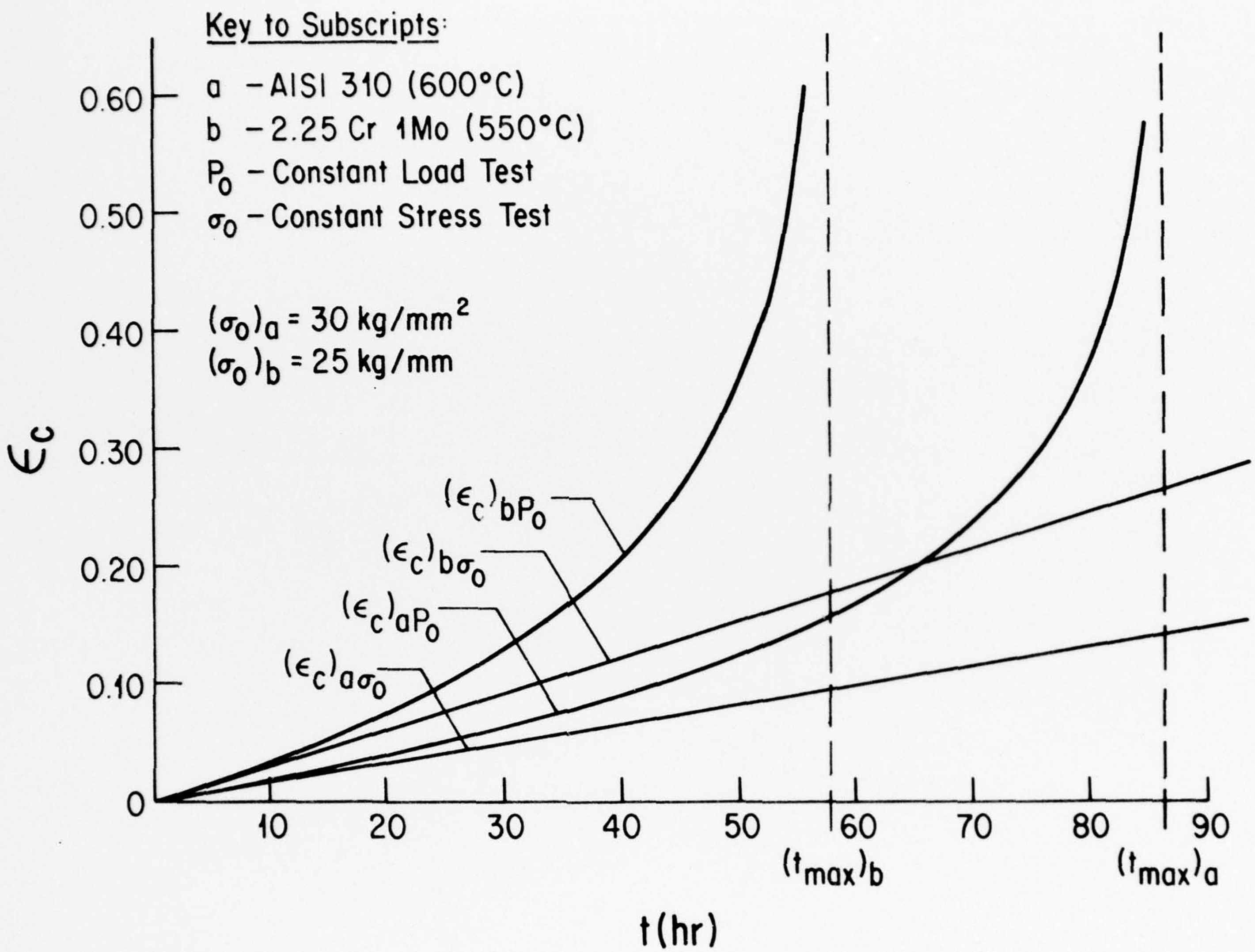

Fig. 1 – Strains at constant load and stress - Large deformation
–Transient creep included approximately in elastic term

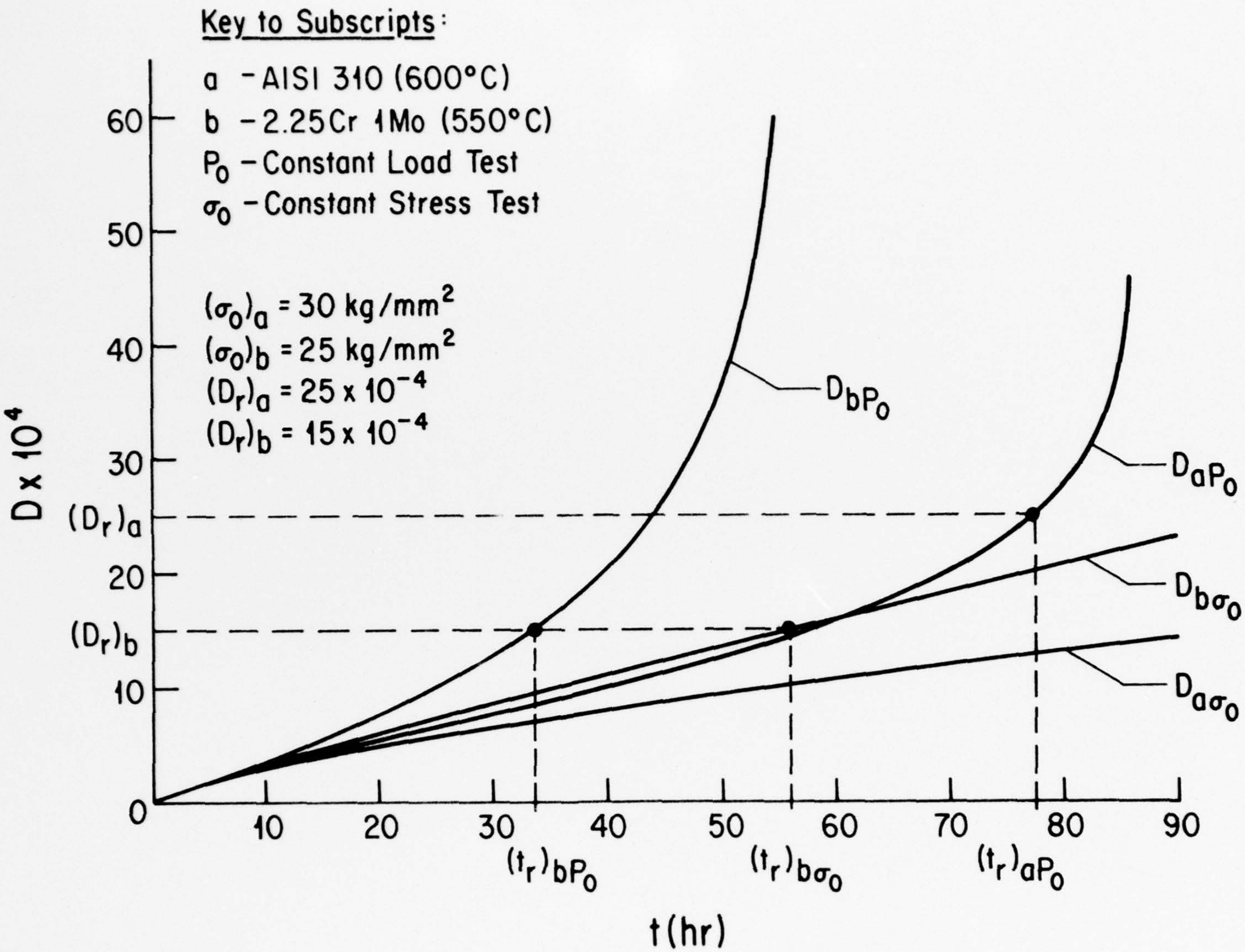

Fig. 2 – Damage at constant load and stress – Large deformation
– Transient creep included approximately in elastic term

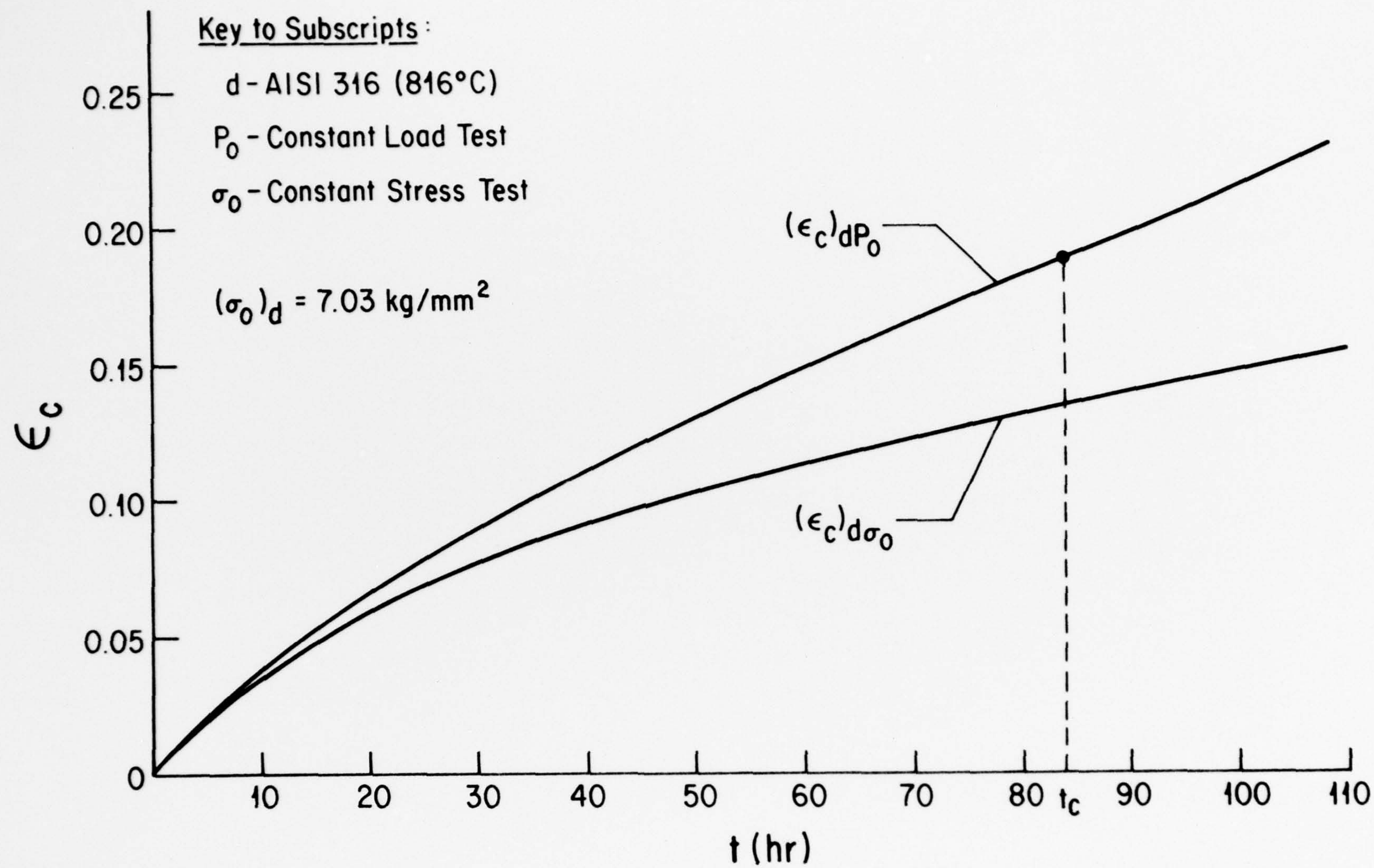

Fig. 3 – Strains at constant load and stress – Small deformation
–Transient creep included as separate term

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A nonlinear damage relation, containing the axial strain history and a time integral over the stress history, is proposed for the case of one-dimensional time dependent tensile stress. Nonlinear steady and transient creep terms are included in the axial strain relation, and elastic and creep Poisson's ratios are introduced into the lateral strain relation. Using these relations, complete damage solutions are obtained for the constant stress rate, step stress, relaxation and constant load tests. Observations are made concerning the associated rupture times.

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